$\boldsymbol{F}$-inverse monoids as algebraic structures in enriched signature

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An inverse monoid is called F-inverse if each class of its minimum group congruence σ has a maximum element with respect to the natural partial order. For instance, free inverse monoids are F-inverse and, consequently, every inverse monoid has an F-inverse cover. When considering a possible computational attack on the problem whether the finitary version of this statement holds, that is, whether each finite inverse monoid has a finite F-inverse cover, Michael Kinyon observed several years ago that the class of all F-inverse monoids forms a variety if the unary operation $a \mapsto a^{\mathtt{m}}$ where $a^{\mathtt{m}}$ is the greatest element in the σ-class of a is added to the set of basic operations. His announcement of this fact at the International Conference on Semigroups (Lisbon, 2018) stimulated us to find the analogue of the Margolis–Meakin expansion within the variety of F-inverse monoids and, in particular, to find transparent models of free F-inverse monoids. Throughout the talk, F-inverse monoids are understood in this extended signature.

For any X-generated group G, we introduce an F-inverse monoid $F(G)$ defined in a way extending the definition of the Margolis–Meakin expansion $M(G)$ of G. Namely, $F(G)$ consists of all pairs (Γ, g) where Γ is any (not necessarily connected) finite subgraph of the Cayley graph $\mathrm{Cay}(G)$ of G having 1 and g as vertices and multiplication is defined by the same rule as in $M(G)$. The F-inverse monoid $F(G)$ turns out to be universal among the X-generated F-inverse monoids F where F/σ is a morphic image of G and, as a consequence, we obtain that $F(G)$ with G being the free group on X is a free F-inverse monoid on X. In the language of categories, our results say that the functor $F\colon G \mapsto F(G)$ from the category of X-generated groups to the category of X-generated F-inverse monoids is an expansion in the sense of Birget and Rhodes, and it is a left adjoint to the functor $\sigma\colon F \mapsto F/\sigma$ going the other way around. In particular, $F(G)$ is an initial object in the category of all X-generated F-inverse monoids F with $F/\sigma = G$.

Universal objects of the same kind are determined also within the subclass of the perfect F-inverse monoids which are defined by the property that σ is a perfect congruence, that is, the set product of σ-classes is a whole σ-class. The class of perfect F-inverse monoids forms a subvariety and is defined, within the variety of F-inverse monoids, by the identity $x^{\mathtt{m}}(x^{-1})^{\mathtt{m}} = 1$.

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