The beautiful simplicity of flawless vigorous groups

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Let $\mathfrak{C}$ be some Cantor space. Our talk will focus on the set of flawless vigorous subgroups of Homeo($\mathfrak{C}$), where we introduce both of these terms here.

We say a group $G \leqslant \mathrm{Aut}(\mathfrak{C})$ is *vigorous* if for any clopen set A and proper clopen subsets B and C of A there is $\gamma \in G$ in the pointwise-stabiliser of $\mathfrak{C}\backslash A$ with $B\gamma \subseteq C$. Being vigorous is similar in impact to some of the conditions proposed by Epstein in his proof that certain groups of homeomorphisms of spaces have simple commutator subgroups (and/or related conditions, as proposed in some of the work of Matui or of Ling). A group $G \leqslant \mathrm{Aut}(\mathfrak{C})$ is *flawless* roughly when its verbal subgroup over *any* non-trivial freely reduced product expression w in k variables, possibly including inverse letters, is the whole group. It is true, for instance, that flawless groups are both perfect and lawless.

Some results here are: vigorous groups are simple if and only if they are flawless, simple vigorous groups are either two-generated by torsion elements, or not finitely generated, and, the class of vigorous simple subgroups of Homeo($\mathfrak{C}$) is fairly broad (it contains many well known groups such as the Higman-Thompson groups $G_{n,r}$, the Brin-Thompson groups nV, Röver's group $V(\Gamma)$, and others of Nekrashevych's 'simple groups of dynamical origin', and, the class is closed under various natural constructions).

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