Monotone maps for partial orders on matrix semigroups

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It was Frobenius who obtained the first result on maps preserving matrix invariants, namely, he described the structure of linear maps preserving the determinant function. Later on there were several extension of this result which are due to Diedonnie, Dynkin, Hua, Schur, and others. In addition to invariants many researchers investigated maps preserving different properties, sets, or relations. Along the same lines, there was intensive study of maps preserving order relations on operator and matrix algebras during the past decades. Among these orders there are relations coming from semigroup theory which date back to Drazin, Hartwig, and Nambooripad. All these notions are widely used in semigroups, linear algebra, operator theory and their applications.

In our talk we introduce a unified approach to define the above order relations via inverses along elements.

We show that in many cases surjective monotone additive maps on the semigroup of matrices are automatically invertible and provide a complete characterization of such maps. Here monotonicity is considered with respect to one of the relations above.