Gaps in probabilities of satisfying some commutator identities

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Let w be a non-trivial word in a free group of rank d and $w: G^d \to G$ a corresponding word map on a finite group G. Let $P_{w=1}(G) = |w^{-1}(1)|/|G|^d$ be the probability that a randomly chosen d-tuple of elements of G evaluates to 1 under the map w. There is an old result of Gustafson stating that if G is a finite non-abelian group, then the commuting probability $P_{[x,y]=1}(G)$ is bounded above by 5/8. Dixon (2004) posed a question whether or not there exists a constant $\eta < 1$ depending on w only such that for every finite group G not satisfying the law w = 1 we have that $P_{w=1}(G) \leq \eta$. We answer the question affirmatively for the 2-Engel word w = [x, y, y] and metabelian word w = [[x, y], [z, w]].

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