Generalizations of Thompson Group V using Synchronizing Automata

SandGAL 2019

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$\mathcal{C}_n = \{0, 1, \ldots n-1\}^\omega$, the infinite words over an n letter alphabet

Let $v_1, v_2, \ldots v_k$ be a list of finite prefixes that 'partition' $\mathcal{C}_n$, i.e. a maximal antichain with respect to lexicographical order.

Also let $u_1, u_2, \ldots u_k$ be another maximal antichain of the same length.

Build the function $f : \mathcal{C}_n \to \mathcal{C}_n$ where $f(v_i w) = u_i w$ for each i. We call f a finite prefix exchange.

Example: $0, 10, 11$ and $01, 1, 00$

Remark

Each finite prefix exchange has infinitely many representations.

Generalized Thompson Groups V_n

The group V_n is the group of all finite prefix exchanges on $\mathcal{C}_n$.

- V_2 is commonly known as V.
- V_n is finitely presentable.
- V_n is simple when n is even.
- V_n has simple derived subgroup of index two when n is odd.
- $V_n \cong V_m$ iff $n = m$.

Subgroup Generating Relations

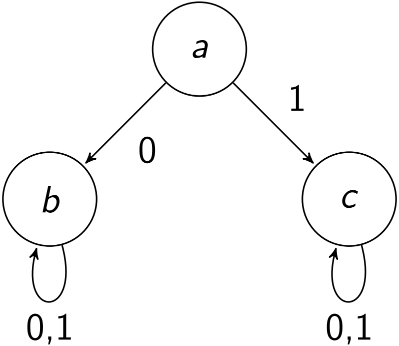
Let $\sim$ be an equivalence relation on $\{0, 1, \ldots n-1\}^*$, the set of finite prefixes. We call $\sim$ subgroup generating when for all finite prefixes α and β,

$$\alpha \sim \beta \text{ if and only if}$$

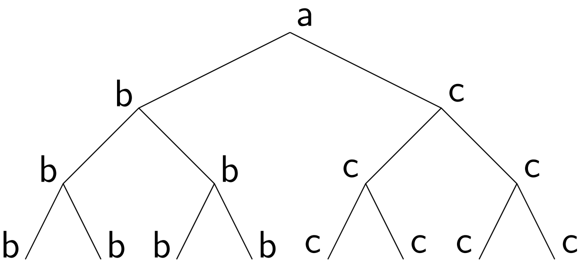
$$\text{for all } i \in \{0, 1, \ldots n-1\}, \alpha i \sim \beta i.$$

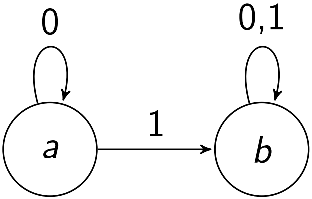
Relation Preserving Subgroups of V_n

We denote the subgroup of V_n that preserves $\sim$ by $V_\sim$.

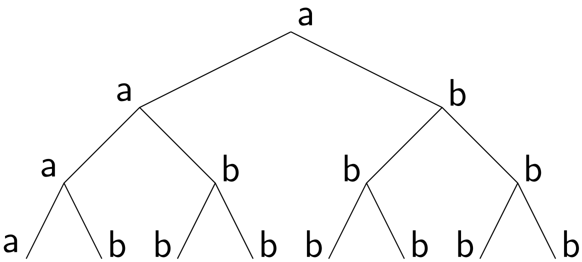


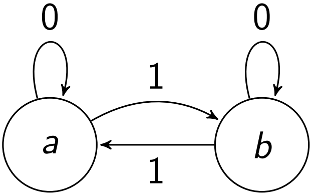
$$V_\sim \cong V \times V$$



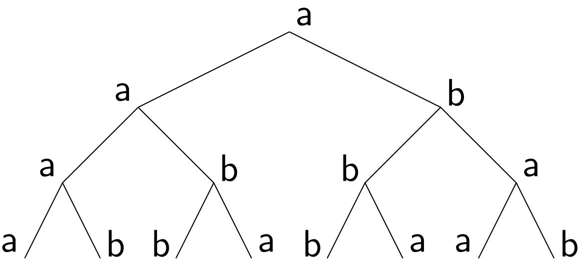


$V_\sim \cong \mathrm{Stab}_V(\overline{0})$





$$V_\sim \cong V$$



Theorem (Bleak, Cameron, Maissel, Navas, Olukoya)

Automorphisms of V_n are precisely conjugations of V_n by homeomorphisms of $\mathcal{C}_n$ induced by bi-synchronizing automata.

Synchronizing Automata

An automaton A is <u>synchronizing</u> (at level k) if every word of length k is a reset word. Furthermore, A is <u>bi-synchronizing</u> if A and A^{-1} are both synchronizing.

Lemma

$V_n^A \leq V_n$ if and only if A is synchronizing.

Theorem (D, Olukoya)

Let A be synchonizing. Then $V_n^A = V_\sim$ where $\sim$ is constructed using A^{-1}.