Structural And Algorithmic Results For HNN Extensions Of Inverse Semigroups

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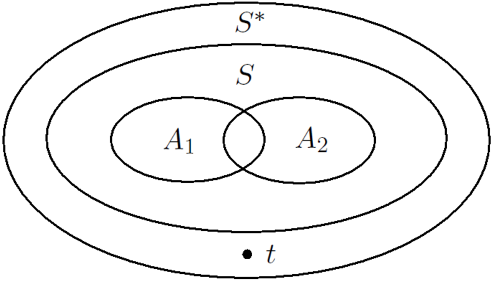
(joint work with Paul Bennett)

HNN Extensions

- Let S be an inverse semigroup and let $\phi : A_1 \to A_2$ be an isomorphism between inverse subsemigroups of S.



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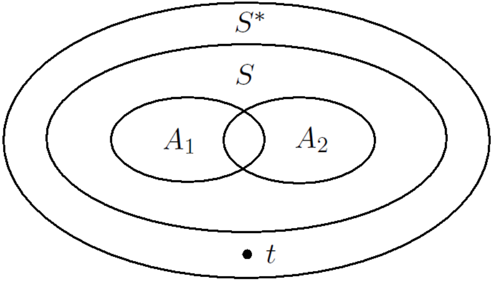

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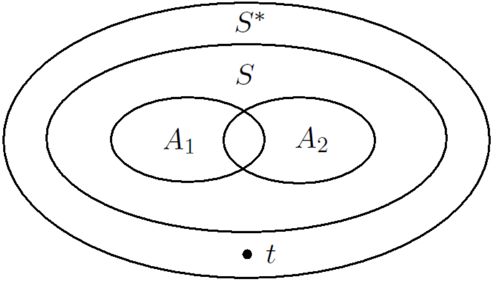
- Yamamura (1997) showed that the HNN extension $S^* = [S; A_1, A_2; \phi]$ contains a copy of S and an element t, with $tt^{-1} = e_1$, $t^{-1}t = e_2$ and $t^{-1}at = (a)\phi$, for all $a \in A_1$.



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A structure theory for the general case still does not exist. The current research attempts to solve this and is joint work with Paul Bennett.

Methods

Schützenberger automata

- connected components of the Cayley graphs
- generalization of Munn trees
- tool to approach algorithmic and structural problems in inverse semigroups
- deterministic inverse word automata

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- one especially useful for the study of structure:

$$G_e \cong Aut(S\Gamma(X, R, e))$$

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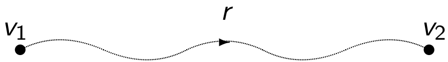

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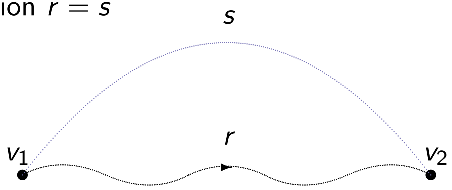

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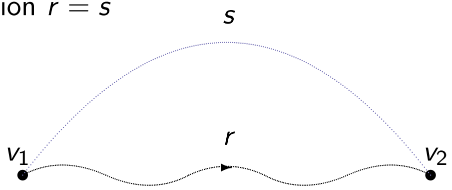
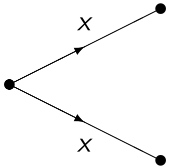


Elementary determination:

-edge folding



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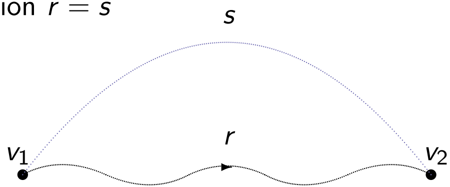

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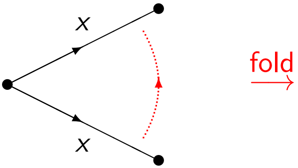

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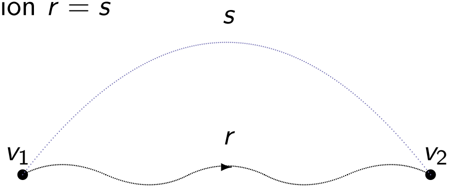

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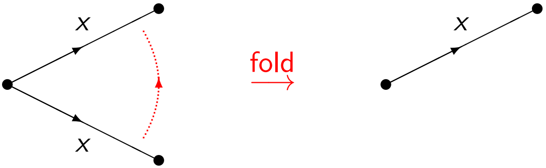

Schützenberger graphs - iterative procedure

In this way we get a directed system of inverse graphs

$$\Gamma_1 \to \Gamma_2 \to \ldots \to \Gamma_i \to \ldots$$

whose directed limit is the Schützenberger graph $S\Gamma(X, R, w)$.

In general, this is:

- infinite
- complicated
- not transparent what is the best way to get to the limit
- never ending
- $\vdots$

Goal: Introduce some order (which we were able to do for some classes of inverse semigroups)

Lower bounded subsemigroups

- We say that A_1 is *lower bounded* in S if $a \geq e$, where $a \in A_1$ and $e \in E(S)$, implies $a \geq f \geq e$, for some $f \in E(A_1)$.

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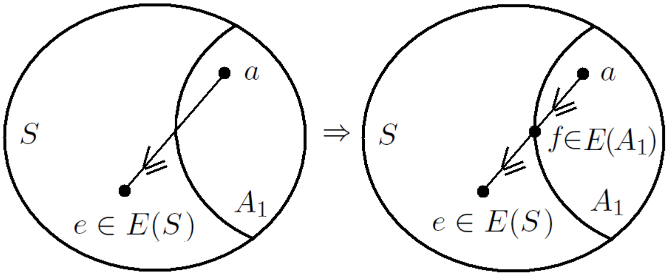

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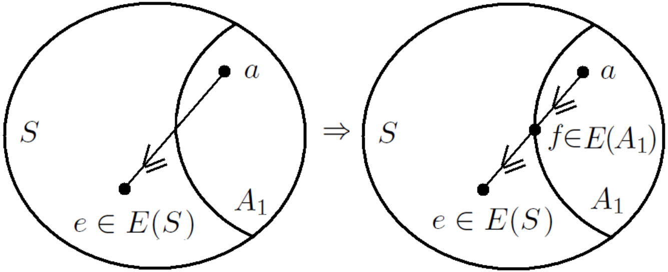

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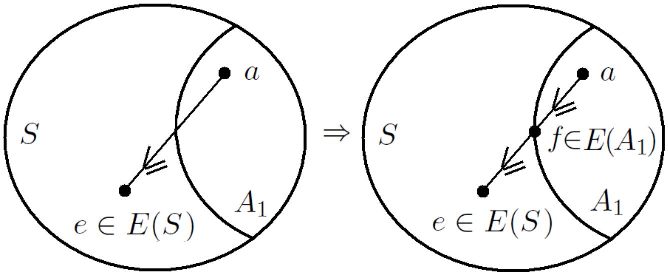
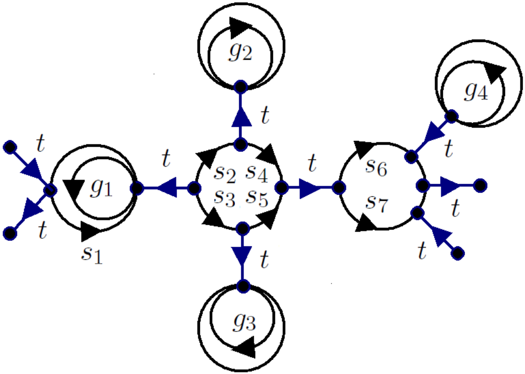


- Similarly, A_2 is *lower bounded* in S if $a \geq e$, where $a \in A_2$ and $e \in E(S)$, implies $a \geq f \geq e$, for some $f \in E(A_2)$.
- We now assume that A_1 and A_2 are lower bounded in S and show how to construct the Schützenberger automata of S^*.

Schützenberger Automata of $S * FIM(t)$

- The underlying graph consists of finitely many maximal connected subgraphs labeled over S or t, called *lobes*.



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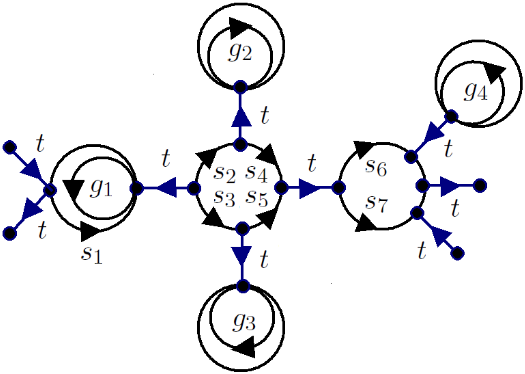

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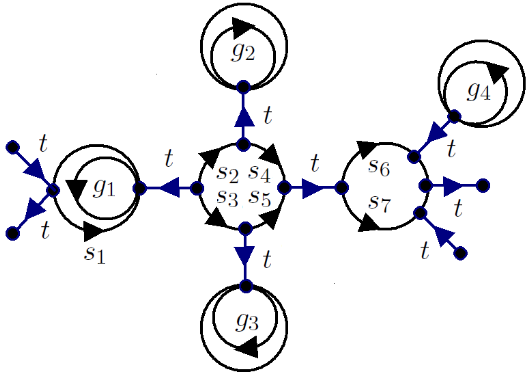

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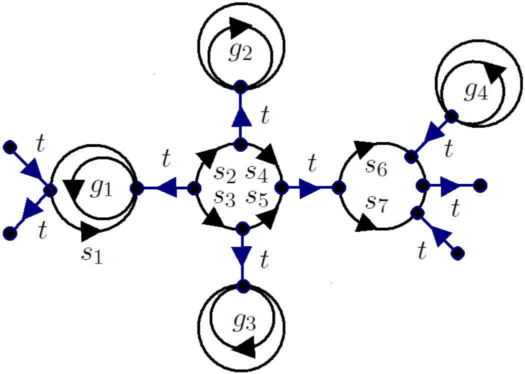
- Black circles are lobes over S, arrows are paths over $S \cup \{t\}$.



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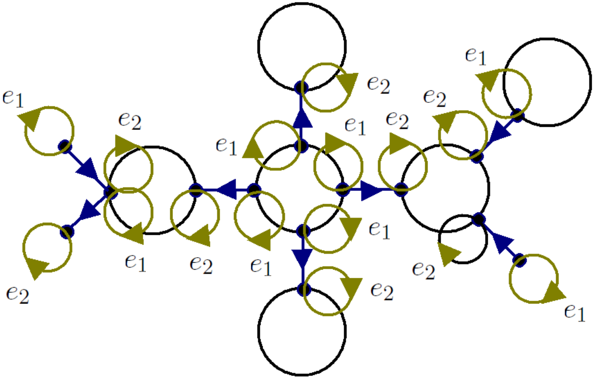

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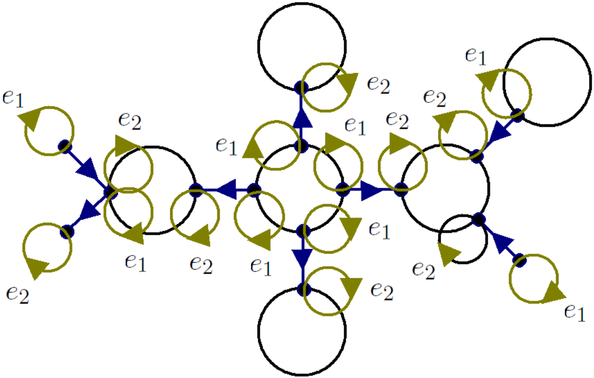

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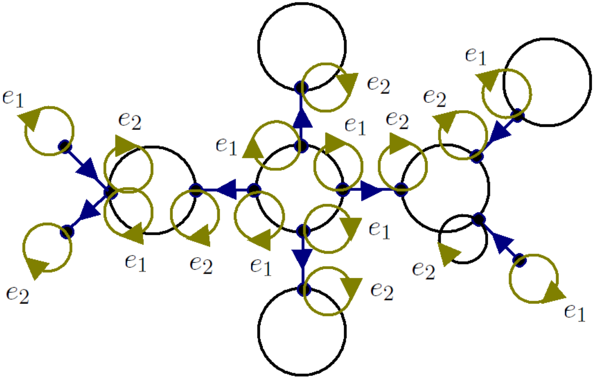
- Then close, relative to $S * FIM(t)$, using the algorithm of Jones et al (1994). The number of black circles may reduce.



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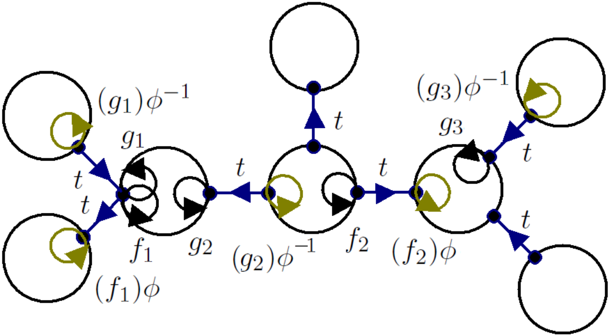

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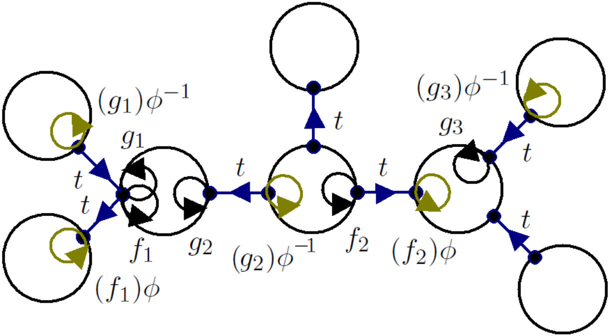

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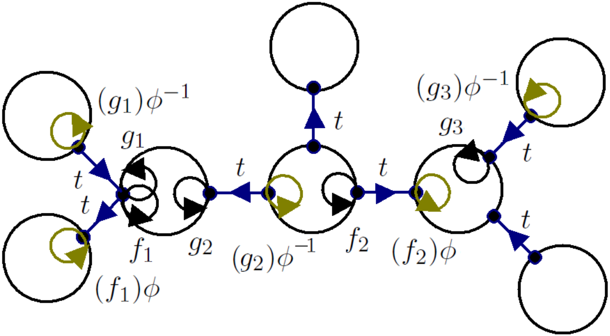
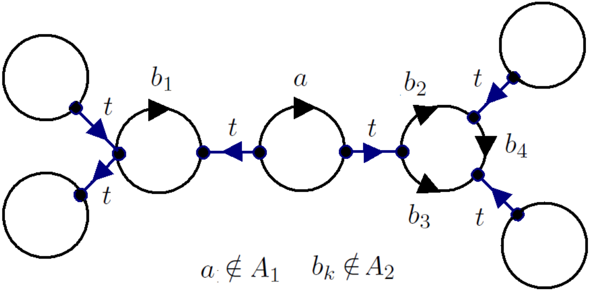


Direct Limit Of Construction 2

- Construction 2 determines a directed system.



$a \notin A_1 \quad b_k \notin A_2$

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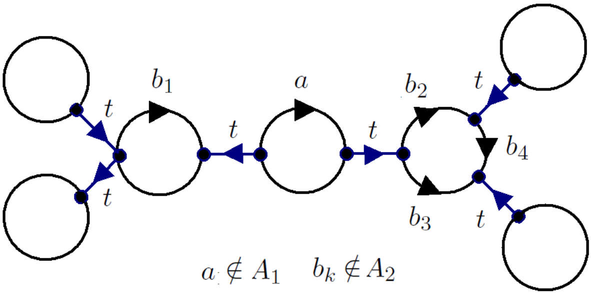

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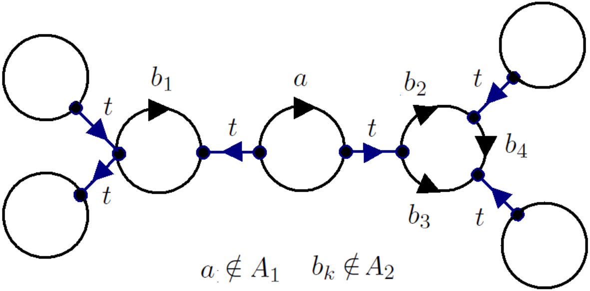

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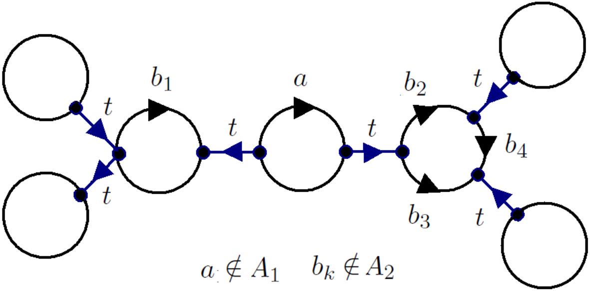

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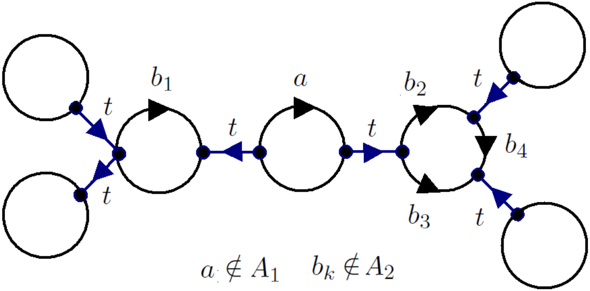
  - The terminal vertices of two t-edges are not connected by a path labeled by any $b \in A_2$.



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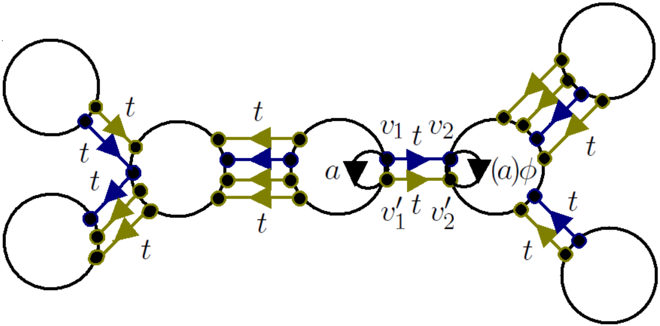
- Sew on a t-edge (green) $v_1' \to^t v_2'$ whenever we have a t-edge $v_1 \to^t v_2$ and a path $v_1 \to^a v_1'$, for some $a \in A_1$, where v_2' is such that we have a path $v_2 \to^{(a)\phi} v_2'$.



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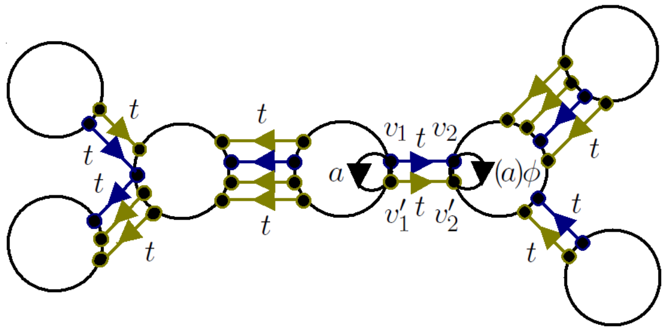

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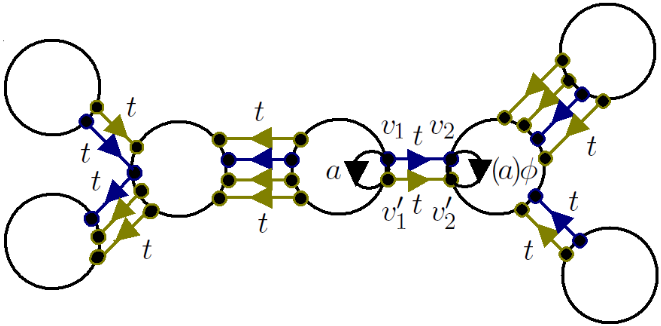
- The new t-edge $v_1' \to^t v_2'$ connects the same black circles as the original t-edge $v_1 \to^t v_2$.
- The resulting automaton has the same *refinements*.



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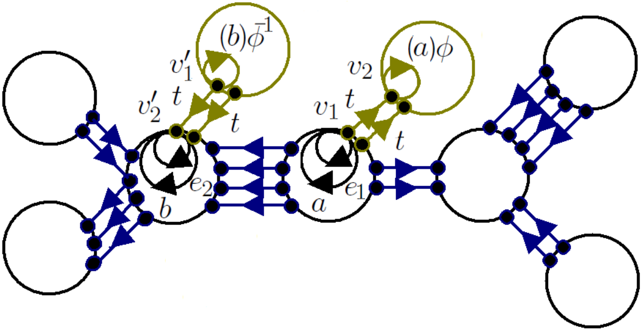

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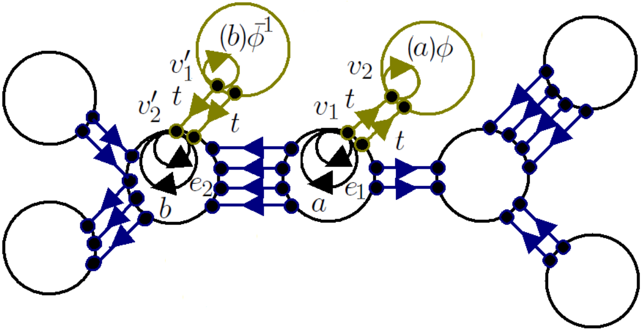

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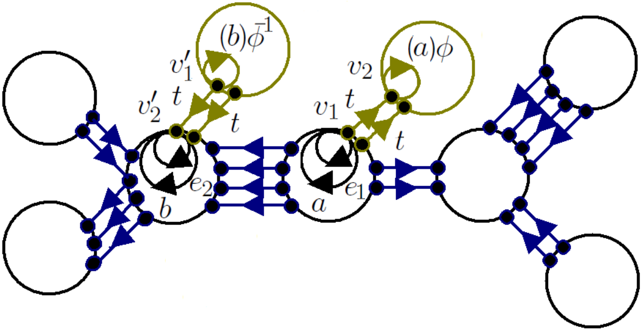
- Close, relative to $S * FIM(t)$, and perform Construction 3.



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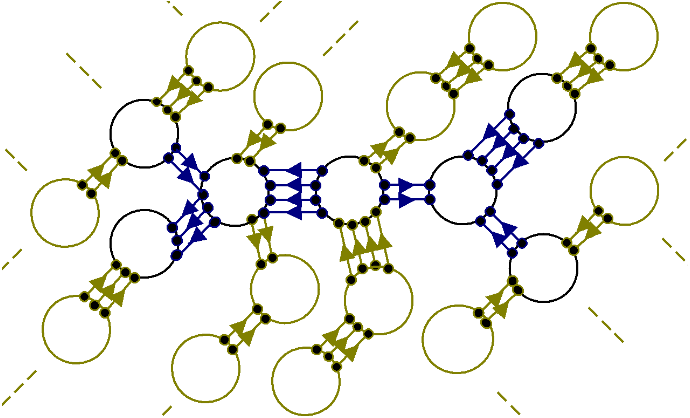

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- Construction 4 determines a directed system.
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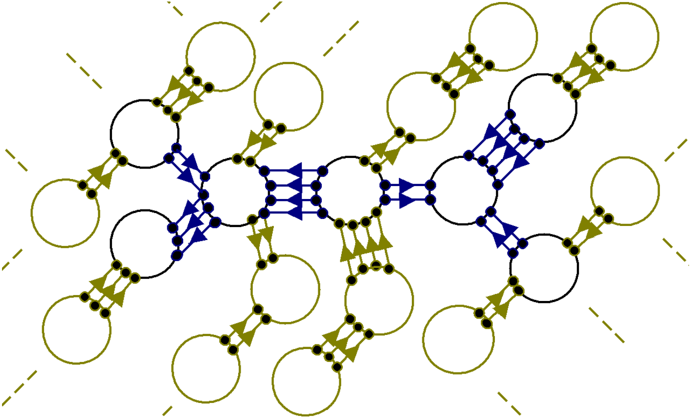

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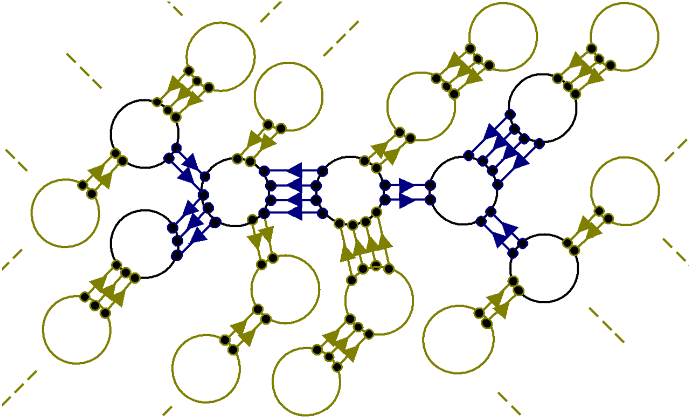
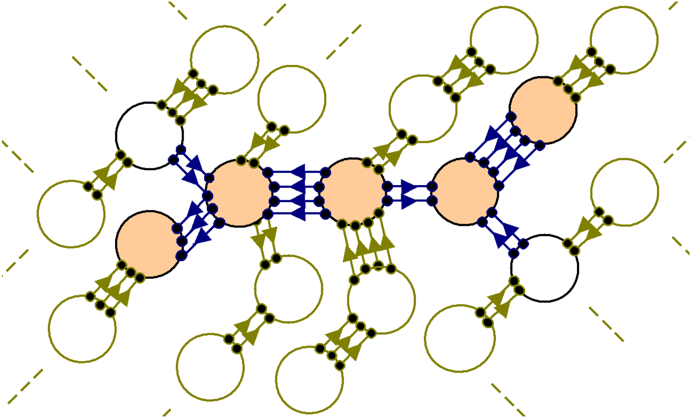
- The direct limit is a Schützenberger automaton of S^*.



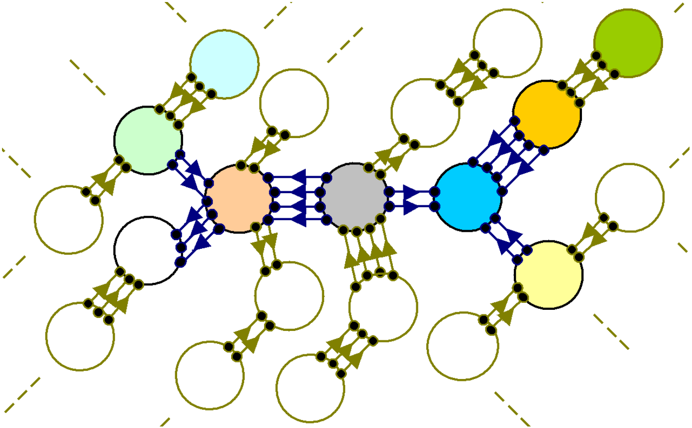
The Host

- Every Schützenberger automaton of S^* has a *host* (orange), defined as a minimal finite-lobe *subgraph* from which the remaining graph *feeds off*, by applications of Construction 4.



The Automorphism Group

- The automorphism group is the automorphism group of the *subgraph* of all hosts, and is also the fundamental group of a graph of groups, defined by the orbits of the hosts.



The Automorphism Group

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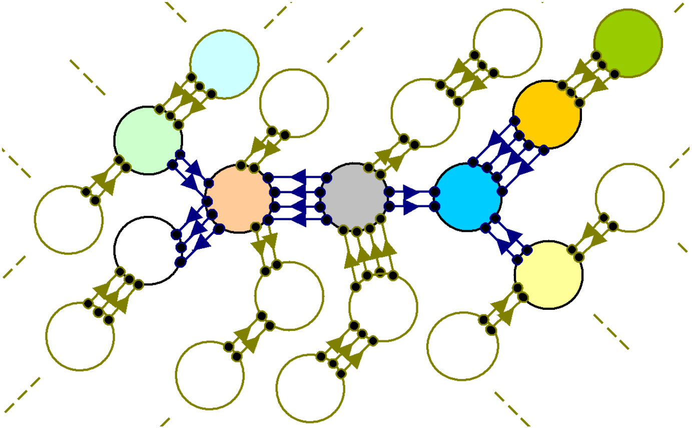
- If there is more than one host then every host is a lobe over S.



Decidable Word Problem For S^*

Results

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- *S has decidable word problem.*
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- *An application of Construction 3 has decidable language.*
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Corollary

If S is finite and finitely presented and A_1, A_2 are lower bounded in S then S^* has decidable word problem.

Special Case: Jajcayová (1997)

HNN's

- $\{a \in A_i : a \geq e\}$ is either empty or has a minimal element, denoted by $f_i(e)$, for all $e \in E(S)$ and $i \in \{1, 2\}$.
- There does not exist an infinite sequence $\{a_k\}$, where $a_k \in E(A_i)$ and $a_k > f_i(ea_k) > a_{k+1}$, for all k.

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Results

- *The Schützenberger automata of S^* can be characterised; the lobes over S are isomorphic to Schützenberger graphs of S.*
- *The maximal subgroups of S^* are either isomorphic to subgroups of S or are the fundamental groups of graphs of groups, defined by the $\mathcal{D}$-classes of A_1, A_2 and S.*
- *S^* is completely semisimple if and only if S is completely semisimple and $\prec \cap \succ_A \subseteq \prec_A$.*
- *If S is free and A_1, A_2 are finitely generated then S^* has decidable word problem.*

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- Define $Z_i = (A_i *_{E(A_i)} M(A_i))/\mu_{A_i}$, for $i = 1, 2$, similarly.
- We have an isomorphism $\pi : Z_1 \to Z_2$ and S is embedded into T_1 and T_2. Put $T = T_1 *_S T_2$.

Constructing A New HNN Extension

Results

If $S^* = [S; A_1, A_2; \phi]$ *is any HNN extension then:*

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If $S^* = [S; A_1, A_2; \phi]$ is any HNN extension then:

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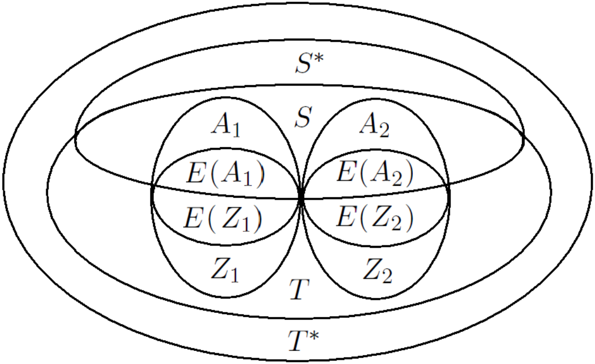
- *Z_1 and Z_2 are lower bounded in T.*
- *S^* is embedded into $T^* = [T; Z_1, Z_2; \pi]$.*



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Special Case: When S Is Finite

Corollaries

- *(Cherubini and Rodaro 2008) S^* has decidable word problem.*
- *(Ayyash 2014) A maximal subgroup of S^* is isomorphc to the fundamental group of a graph of groups, defined by the $\mathcal{D}$-classes of S, A_1 and A_2, or is a homomorphic image of a subgroup of S.*
- *(Ayyash 2014) S^* is completely semisimple if and only if S is completely semisimple and $\prec \cap \succ_A \subseteq \prec_A$.*

Thank you!