Some properties of the metanorm of a group

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Norm

1. If G is a group, the *norm* $N(G)$ of G is the intersection of the normalizers of all subgroups of G.
2. The concept of norm has been introduced by R. Baer in 1934.
3. $G = N(G)$ if and only if all subgroups of G are normal (i.e. G is a Dedekind group)
4. If $N(G)$ is not abelian, then G is periodic (Baer).
5. If $N(G)$ is not periodic, then $N(G) = Z(G)$ (Baer).
6. $N(G) \leqslant Z_2(G)$ (Schenkman, 1960).
7. $x \in N(G)$ if and only if the inner automorphism induced by x is a power automorphism.

   Schenkman's result is also a consequence of the fact that every power automorphism is central (Cooper, 1968).

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$\mathfrak{X}$- norm

Let $\mathfrak{X}$ be a class of groups

1. The $\mathfrak{X}$-norm of G is the intersection of all normalizers of subgroups of G which are not in $\mathfrak{X}$ (A. Ballester-Bolinches, J. Cossey, Z. Zhang, 2014).
2. The norm coincides with the $\mathfrak{I}$-norm, where $\mathfrak{I}$ is the class of all trivial groups.
3. The $\mathfrak{T}$-norm, where $\mathfrak{T}$ is the class of all periodic groups, has been studied by M. De Falco, F. de Giovanni, C. Musella, Y.P. Sysak (2018).
4. Survey on properties of the various relevant norms: M.G. Drushlyak, T.D. Lukashova, F.M. Lyman, Generalized norms of groups, Algebra Discrete Math. 22 (2016) 48–81.

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Metahamiltonian groups

- G is called *metahamiltonian* if every non-abelian subgroup of G is normal.
- Metahamiltonian groups were introduced and studied in a series of papers by G.M. Romalis and N.F. Sesekin (1966–1970).
- A group G is said to be *locally graded* if every finitely generated non-trivial subgroup has a proper subgroup of finite index
- If G is a locally graded metahamiltonian group, G' finite and has prime-power order (the result has been proved by Romalis and Sesekin for soluble groups).
- If G is a locally graded metahamiltonian group and $|G'|$ is a power of the prime number p, we will call p *the characteristic of* G.
- The structure of soluble metahamiltonian groups has been described also by N.F. Kuzennyi, N.N. Semko (1983–1990).

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Theorem (N.F. Kuzennyi, N.N. Semko)
Let G be a locally graded metahamiltonian non-nilpotent group.

1. If G is metabelian, then $\exists K \leqslant G$ such that $G = K \ltimes G'$, $K/C_K(G')$ is cyclic and for each $y \in K \setminus C_K(G')$, G' is a minimal $\langle y \rangle$-invariant subgroup.
2. If G is not metabelian, G is periodic and if p is the characteristic of G, then $|G''| = p$, $|G'| = p^3$ and G/G' has no elements of oder p. Moreover, $G'' \leqslant Z(G)$, G'/G'' is an eccentric chief factor of G, and $G/C_G(G'/G'')$ is cyclic.

Moreover, it follows from other results of Kuzennyi and Semko that any nilpotent metahamiltonian group is metabelian.

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Influence of the metanorm on the group structure

Theorem (MDF, F. de Giovanni, L.A. Kurdachenko, C. Musella)
Let G be a locally finite group such that $M = M(G)$ is not nilpotent.
If $M(G)$ is metabelian, then G is metahamiltonian, unless $|M'| = p^2$ (where p is the caharacteristic of $M(G)$).

Theorem (MDF, F. de Giovanni, L.A. Kurdachenko, C. Musella)
Let G be a locally finite group.
If $M(G)$ is not metabelian, then G contains an abelian subgroup whose index is finite and divides $p(p-1)(p^2-1)$, where p is the caharacteristic of $M(G)$ and every p-Sylow of G is nilpotent of class at most 2.

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Corollary (MDF, F. de Giovanni, L.A. Kurdachenko, C. Musella)
Let G be a locally polycyclic-by-finite group such that $M = M(G)$ is not nilpotent.
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It was shown by Kuzennyi and Semko that in a soluble metahamiltonian group G every non-abelian subgroup contains G'.

Proposition (MDF, F. de Giovanni, L.A. Kurdachenko, C. Musella)
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$G/M(G)$ and G'

- A celebrated theorem of I.Schur states that if $G/Z(G)$ is finite, then G' is finite.
- If $G/Z(G)$ is polycyclic-by-finite, then G' is polycyclic-by-finite.

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It follows from Baer's result on groups with non-periodic norm, that $[N(G), G]$ is always periodic.

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Metanorm and upper central series

Theorem (MDF, F. de Giovanni, L.A. Kurdachenko, C. Musella)
Let G be a locally graded group, and put $M = M(G)$. Then:

1. $M'' \leqslant Z(G)$
2. if M is not nilpotent, then $M/M' \leqslant Z(G/M')$

Corollary (MDF, F. de Giovanni, L.A. Kurdachenko, C. Musella)
Let G be a locally graded group.
If $M = M(G)$ is not nilpotent, then $Z_2(M) \leqslant Z_2(G)$.

Proof – $M/M' \leqslant Z(G/M') \;\Rightarrow\; [M, G] \leqslant M'$.
$\Rightarrow\; [Z_2(M), G] \leqslant Z_2(M) \cap M'$.
Put $L = Z_2(M) \cap M'$.
M'/M'' is an eccentric chief factor of $M \;\Rightarrow$
$LM'' < M' \;\Rightarrow L \leqslant M''$
$\Rightarrow\; L \leqslant Z(G) \;\Rightarrow\; [Z_2(M), G] \leqslant Z(G)$
$[Z_2(M), G, G] = \{1\}$. □

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Let G be a locally nilpotent group.
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